

Sudakov suppression in azimuthal spin asymmetries

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It is shown that transverse momentum dependent azimuthal spin asymmetries suffer from suppression due to Sudakov factors, in the region where the transverse momentum is much smaller than the large energy scale Q^2 . The size and Q^2 dependence of this suppression are studied numerically for two such asymmetries, both arising due to the Collins effect. General features are discussed of how the fall-off with Q^2 is affected by the nonperturbative Sudakov factor and by the transverse momentum weights and angular dependences that appear in different asymmetries. For a subset of asymmetries the asymptotic Q^2 behavior is calculated analytically, providing an upper bound for the decrease with energy of other asymmetries. The effect of Sudakov factors on the transverse momentum distributions is found to be very significant already at present-day collider energies. Therefore, it is essential to take into account Sudakov factors in transverse momentum dependent azimuthal spin asymmetries.

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I. INTRODUCTION

In this article we will study the effects of Sudakov factors in transverse momentum dependent azimuthal spin asymmetries, like the Collins effect asymmetry of Ref. [1]. We will demonstrate explicitly that such asymmetries suffer from suppression due to these Sudakov factors, in the region where the transverse momentum is much smaller than the large energy scale Q^2 . This Sudakov suppression stems from soft gluon radiation and increases with energy. It implies that tree level estimates of such asymmetries tend to overestimate magnitudes and increasingly so with rising energy. In this paper, the Q^2 dependence of the suppression due to Sudakov factors will be studied numerically in two examples which are relevant for present-day experimental studies. In addition, the asymptotic Q^2 dependence of an important subset of asymmetries is calculated analytically, providing an upper bound for the decrease with energy of other asymmetries.

The first example we consider is a Collins effect driven $\cos(2\phi)$ asymmetry in electron-positron annihilation into two almost back-to-back pions [2,3], which in principle can be determined from existing LEP data. This asymmetry allows for a determination of the Collins effect fragmentation function, which in turn would be useful for the extraction of the transversity distribution function h_1 . The latter can be done via the single spin asymmetry in semi-inclusive Deep Inelastic Scattering (DIS) which was originally proposed by Collins [1] –our second example–. In Ref. [1] Collins already remarks that the Sudakov factors will have the effect of diluting this single spin asymmetry due to broadening of the transverse momentum distribution by soft gluon emission. We will study this effect in a quantitative way to gain insight into the parametric dependences of such Sudakov suppression.

For this we will follow the recent analysis [4] of a helicity non-flip double transverse spin asymmetry in vector boson production in hadron-hadron scattering. That asymmetry is for instance relevant for the polarized proton-proton collisions to be performed at BNL-RHIC. In Ref. [4] the effect of the Sudakov factors compared to the tree level asymmetry expression was numerically estimated. It was shown that the inclusion of Sudakov factors cause suppression by at least an order of magnitude compared to the tree level result at scales around M_W or M_Z . Moreover, the suppression increases with energy approximately as $Q^{0.6}$ (in the studied range of roughly 10 – 100 GeV). The conclusion was that the Sudakov suppression together with a kinematic suppression (due to explicit lightcone momentum fractions that appear in the prefactor) imply that the asymmetry will be negligible for Z or W production and is interesting only at much lower energies.

Similarly, Sudakov suppression will turn out to be an important issue for the actual determination of the above mentioned $\cos(2\phi)$ asymmetry from LEP data (or in general at high energy e^+e^- colliders). Due to the

lack of knowledge of the nonperturbative Sudakov factor in the case of electron-positron annihilation into almost back-to-back hadrons, solid numerical conclusions about the size and Q^2 dependence of the suppression cannot be drawn. Nevertheless, one can draw conclusions about what determines the Q^2 dependence of the transverse momentum distribution of the asymmetries and about the size of the suppression for generic nonperturbative Sudakov factors. For that purpose comparisons to tree level are most instructive, since the evolution of the often unknown distribution and fragmentation functions themselves does not affect the relative Sudakov suppression compared to tree level.

The two examples to be investigated here –the above mentioned $\cos(2\phi)$ asymmetry and the original Collins $\sin(\phi)$ asymmetry in semi-inclusive DIS–, have different transverse momentum weights and angular dependences, producing different Sudakov suppression effects. General conclusions about how these properties affect the Q^2 dependence can be drawn. In general, a larger power of transverse momentum in the weight of an asymmetry implies larger suppression. Both the numerical calculations at realistic collider energies and the asymptotic behavior of asymmetries exhibit this feature.

Before going into the details of the specific examples, we will first give an overview of the issues regarding factorization theorems for transverse momentum dependent cross sections, in the region where the transverse momentum is very small compared to the hard scale in the process [5–7]. This will provide the theoretical justification of the factorized asymmetry expressions that will be derived.

The outline of this paper is as follows. In section II we will discuss the essentials of factorization theorems for the Drell-Yan process [7] (relevant for the double transverse spin asymmetry studied in Ref. [4]), which are in fact completely analogous to the case of back-to-back jets in electron-positron annihilation [5,6] and semi-inclusive DIS (SIDIS). Following the approach of Ref. [4], we will then study the $\cos(2\phi)$ asymmetry in electron-positron annihilation (section III) and the Collins $\sin(\phi)$ asymmetry in semi-inclusive DIS (section IV). In section V we will discuss the asymptotic behavior of the Sudakov suppression for certain subsets of asymmetries.

II. FACTORIZATION AND TRANSVERSE MOMENTUM

For definiteness, we will consider here the Drell-Yan process ($H_1 + H_2 \rightarrow \ell + \bar{\ell} + X$) and discuss what is known about the factorization of this process. First of all, it is well-known that a local operator product expansion (OPE) cannot be applied, since the process receives relevant contributions off the light cone [8], i.e. from matrix elements like

$$\sum_X \langle P_1, P_2 | J_\mu(\xi) | X \rangle \langle X | J_\nu(0) | P_1, P_2 \rangle \quad \text{with } \xi^2 \neq 0. \quad (1)$$

Therefore, one usually considers nonlocal operators and what is called a “working redefinition of twist” [8], which means the lowest value of t in $1/Q^{t-2}$ at which a function can contribute to the cross section (Q is the invariant mass of the lepton pair). The nonlocal operators appear in the leading twist factorization theorem

$$\frac{d\sigma}{d\Omega dx_1 dx_2} = \sum_{a,b} \int_{x_1}^1 dx \int_{x_2}^1 d\bar{x} \Phi^a(x) H^{ab}(x, \bar{x}; Q^2) \bar{\Phi}^b(\bar{x}), \quad (2)$$

where a, b are flavor indices and $H^{ab}(x, \bar{x}; Q^2)$ is the hard (partonic) part of the scattering, which is a function of the hard scale Q^2 and the lightcone momentum fractions x, \bar{x} only. The correlation functions Φ ($\bar{\Phi}^a = \Phi^{\bar{a}}$) describe the soft (nonperturbative) physics and are given by nonlocal operator matrix elements

$$\Phi_{ij}(x) \equiv \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | \bar{\psi}_j(0) \mathcal{L}^+[0, \lambda n_-] \psi_i(\lambda n_-) | P, S \rangle, \quad (3)$$

where the path-ordered exponential,

$$\mathcal{L}^+[0, \lambda n_-] = \mathcal{P} \exp \left(-ig \int_0^\lambda d\eta A^+(\eta n_-) \right), \quad (4)$$

renders the matrix element color gauge invariant. The hadronic state $|P, S\rangle$ is determined by the hadron momentum P and spin vector S . Also, we have suppressed the factorization scale dependence in Eq. (2).

Secondly, if one considers the cross section differential in the transverse momentum \mathbf{q}_T of the lepton pair, then for the case of $\mathbf{q}_T^2 \equiv Q_T^2 \ll Q^2$ collinear factorization, like in Eq. (2), does not hold. Instead, the leading twist factorization takes the form [5–7,9,10]

$$\frac{d\sigma}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \sum_{a,b} \int_{x_1}^1 dx \int_{x_2}^1 d\bar{x} \int d^2\mathbf{k}_T d^2\mathbf{p}_T \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{b}\cdot(\mathbf{p}_T+\mathbf{k}_T-\mathbf{q}_T)} \times \Phi^a(x, \mathbf{k}_T) H^{ab}(x, \bar{x}; Q) e^{-S(b,Q)} \bar{\Phi}^b(\bar{x}, \mathbf{p}_T) + Y(x_1, x_2, Q, Q_T). \quad (5)$$

The correlation function $\Phi^a(x, \mathbf{k}_T)$ is now also a function of transverse momentum, the factor $e^{-S(b,Q)}$ is the so-called Sudakov form factor and the factor $Y(x_1, x_2, Q, Q_T)$ becomes important only when $Q_T \sim Q$. Here we use a different notation than in e.g. Refs. [5–7,9,10], but to make a connection with for instance the notation of Ref. [5] we note that $\Phi(x, \mathbf{k}_T)$ corresponds to the Fourier transform of $\hat{\mathcal{P}}_{A/i}(x_A, \mathbf{b}) \mathcal{P}_A/2$ and the e^{-S} terms correspond to each other.

The factor $Y(x_1, x_2, Q, Q_T)$ is present to retrieve collinear factorization when Q_T is a hard scale itself, i.e. $Q_T \sim Q$. We will neglect the Y term from here on, since we will only be interested in the region of $Q_T^2 \ll Q^2$. We will now comment on the other terms in the above factorization theorem and explain the additional restrictions we will impose in the limit $Q_T^2 \ll Q^2$.

The factor $e^{-S(b,Q)}$ is the so-called Sudakov form factor, which arises due to exponentiation of soft gluon contributions. This is in contrast to inclusive cross sections, like Eq. (2), in which there is a cancellation of soft gluon contributions. At values $b^2 = \mathbf{b}^2 \ll 1/\Lambda^2$, the Sudakov form factor is perturbatively calculable and is of the form

$$S(b, Q) = \int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A(\alpha_s(\mu)) \ln \frac{Q^2}{\mu^2} + B(\alpha_s(\mu)) \right], \quad (6)$$

where $b_0 = 2 \exp(-\gamma_E) \approx 1.123$ (we choose the usual constants $C_1 = b_0, C_2 = 1$). One can expand the functions A and B in α_s and the first few coefficients are known, e.g. [11,12]. In order to obtain a first estimate of the effect of including the Sudakov factor we will consider only the leading contribution, i.e. take into account only the first term in the expansion of A : $A^{(1)} = C_F/\pi$, which is the same for unpolarized as well as polarized scattering. This leads to the expression [13]

$$S(b, Q) = -\frac{16}{33 - 2n_f} \left[\log \left(\frac{b^2 Q^2}{b_0^2} \right) + \log \left(\frac{Q^2}{\Lambda^2} \right) \log \left[1 - \frac{\log(b^2 Q^2/b_0^2)}{\log(Q^2/\Lambda^2)} \right] \right]. \quad (7)$$

We will take for the number of flavors $n_f = 5$ and also $\Lambda_{QCD} = 200 \text{ MeV}$.

The correlation function $\Phi^a(x, \mathbf{k}_T)$ is now defined as [9,14]

$$\Phi(x, \mathbf{k}_T) \equiv \int dk^- \left[\int \frac{d^4x}{(2\pi)^4} e^{ik \cdot x} \langle P, S | \bar{\psi}(0) \mathcal{L}^+[0, -\infty] \mathcal{L}^+[-\infty, x] \psi(x) | P, S \rangle \right] \Big|_{k^+ = xP^+, \mathbf{k}_T}, \quad (8)$$

with for each colored field a path-ordered exponential like

$$\mathcal{L}^+[-\infty, x] = \mathcal{P} \exp \left(-ig \int_{-\infty}^{x^-} dy^- A^+(y) \right) \Big|_{y^+ = x^+, \mathbf{y}_T = \mathbf{x}_T}. \quad (9)$$

The correlation function $\Phi^a(x, \mathbf{k}_T)$ also has a factorization scale dependence ($\mu = b_0/b$), which is linked to that of the hard scattering part H .

The correlation function $\Phi^a(x, \mathbf{k}_T)$ contains the nonperturbative dependence on the transverse momentum, which cannot be calculated perturbatively and should be fitted to experiment. In for example Refs. [5–7] it is explained how these functions can be replaced by the ordinary parton distribution functions $\Phi^a(x)$, by introduction of a b -regulator, e.g. the usual cut-off b_{\max} , via $b_* = b/\sqrt{1 + b^2/b_{\max}^2}$, and by introducing a nonperturbative Sudakov factor S_{NP} . Schematically, this proceeds as follows. The first term in Eq. (5) can be written as an overall \mathbf{b} integration of an integrand called $\tilde{W}(b)$,

$$\frac{d\sigma}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \int_{x_1}^1 dx \int_{x_2}^1 d\bar{x} \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_T} \tilde{W}(b) + Y. \quad (10)$$

This function $\tilde{W}(b)$ can be trivially rewritten as $\tilde{W}(b_*) \times (\tilde{W}(b)/\tilde{W}(b_*))$. The first term ($\tilde{W}(b_*)$) can be calculated within perturbation theory since $\alpha_s(b_0/b_*)$ is always small (we will take the usual $b_{\max} = 0.5 \text{ GeV}^{-1}$, hence $\alpha_s(b_0/b_*) \gtrsim 2 \lesssim 0.3$) and the second term can be shown to be of the form

$$\frac{\tilde{W}(b)}{\tilde{W}(b_*)} = \exp(-\ln(Q^2/Q_0^2)g_Q(b) - g_A(x, b) - g_B(\bar{x}, b)) \equiv \exp(-S_{NP}(b)). \quad (11)$$

The functions g are not calculable in perturbation theory and need to be fitted to experiment. The functions $g_A(x, b)$ and $g_B(\bar{x}, b)$ parameterize the intrinsic transverse momentum of the functions $\tilde{\Phi}^a(x, b)$ and $\tilde{\Phi}^b(\bar{x}, b)$ (in the polarized case $\tilde{\Phi}^a$ is actually a function of \mathbf{b} , see further comments below). It will be important later on that the functions g_A and g_B are independent of the scale Q . Finally, a lightcone expansion allows to express $\tilde{\Phi}^a(x, b_*)$ in terms of $\Phi^a(x)$.

We will consider $\tilde{\Phi}^a(x, \mathbf{b}_*)\tilde{\Phi}^b(\bar{x}, \mathbf{b}_*)H(x, \bar{x}; Q)$ only to lowest order in α_s . The reason is that the perturbative tail of $\tilde{\Phi}^a(x, \mathbf{b}_*)$ is not known for the Collins function. Because of the unknown size of the distribution and/or fragmentation functions appearing in the asymmetries, the magnitude of the asymmetries cannot be estimated. Therefore, here we will be interested in the effect of soft gluons on the transverse momentum dependence of the asymmetries, which determines the Q^2 dependence of the Q_T dependence. Although the hard scattering part is restricted to tree level, one can draw conclusions on the size of the effect of Sudakov factors by comparison to the tree level result, because both results will receive the same corrections to the hard part. Corrections to the hard scattering part can be easily inserted with hindsight (for a one-loop example cf. Ref. [5], pages 432 and 433). As said we are interested in the Q^2 evolution of the Q_T dependence of the asymmetries. This Q^2 dependence arises in a nontrivial way via Bessel functions, such that a numerical study is warranted.

In Refs. [5–7] unpolarized scattering is considered, where only dependence on the length of \mathbf{b} appears. Here we will consider polarized scattering and the above sketched procedure of Refs. [5–7] to factor a nonperturbative part from $\tilde{W}(b)$, can also be applied after $\Phi^a(x, \mathbf{k}_T)$ is parameterized in terms of functions of \mathbf{k}_T^2 . Explicit factors of $\mathbf{k}_T \cdot \mathbf{S}_T$ or \mathbf{k}_T can be included in the hard scattering part H .

The above factorization theorem for the Drell-Yan process provides the justification of the factorized expression for the double transverse spin asymmetry studied in Ref. [4]. In Ref. [5,6] an analogous factorization theorem had been discussed earlier for back-to-back jets in electron-positron annihilation, which comes down to a replacement of $\Phi(x, \mathbf{k}_T) \rightarrow \Delta(z, \mathbf{k}_T)$ –the fragmentation correlation function–. In Ref. [1] it is argued that a similar factorization theorem holds for semi-inclusive DIS. Therefore, factorization theorems analogous to the one discussed above (differing only by obvious replacements), provide the theoretical foundation for the examples to be considered below.

It is interesting to see what happens if one takes tree level everywhere in the above factorized expression Eq. (5) (ignoring the Y term). At tree level one finds the reduction

$$\int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{b} \cdot (\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T)} e^{-S(\mathbf{b})} \rightarrow \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T), \quad (12)$$

such that one arrives at the expression used by Ralston and Soper for the hadron tensor [15,16,4]

$$\mathcal{W}^{\mu\nu} = \frac{1}{3} \int d^2\mathbf{p}_T d^2\mathbf{k}_T \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T) \text{Tr}(\Phi(x_1, \mathbf{p}_T) V_1^\mu \bar{\Phi}(x_2, \mathbf{k}_T) V_2^\nu) \Big|_{p^+, k^-} + \left(\begin{matrix} q \leftrightarrow -q \\ \mu \leftrightarrow \nu \end{matrix} \right). \quad (13)$$

The vertex V_i^μ can be either the photon, Z or W boson vertex. In the above unpolarized case $\Phi(x, \mathbf{p}_T)$ is parameterized as $\frac{1}{2}f_1(x, \mathbf{p}_T^2)\mathcal{P}$, where f_1 is the parton momentum distribution function. Collins [17] has shown that polarization does not affect the factorization theorems and therefore, we conclude that the tree level formalism of Ralston and Soper Eq. (13) is in accordance with the factorization theorem Eq. (5) also in the case when polarization is taken into account. Clearly, Eq. (13) is only applicable for Q_T values of the order of the intrinsic transverse momentum. To extend the formalism of Ralston and Soper to larger values of Q_T , but still under the restriction of $Q_T^2 \ll Q^2$, one can use the above factorized expression Eq. (5) (without the Y term) beyond tree level.

III. UNPOLARIZED ASYMMETRY IN ELECTRON-POSITRON ANNIHILATION

In this section we investigate a $\cos(2\phi)$ asymmetry in the process of electron-positron annihilation into two almost back-to-back pions $e^+e^- \rightarrow \pi^+\pi^-X$. This process is very similar to the Drell-Yan process, so we can employ the same expression as given in Eq. (5), but with the correlation functions Φ , replaced by the fragmentation correlation function [18]

$$\Delta_{ij}(k) = \sum_X \int \frac{d^4 z}{(2\pi)^4} e^{ik \cdot z} \langle 0 | \psi_i(x) \mathcal{L}^-[x, \infty] | P, S; X \rangle \langle P, S; X | \mathcal{L}^-[\infty, 0] \bar{\psi}_j(0) | 0 \rangle, \quad (14)$$

which is used to define transverse momentum dependent fragmentation functions.

The asymmetry requires observing the transverse momentum of the vector boson compared to the two pions. The tree level asymmetry expression for the $\cos(2\phi)$ asymmetry in the process $e^+e^- \rightarrow \pi^+\pi^-X$ was discussed in Refs. [2,3], to which we refer for details. Here we shortly repeat the essentials. The asymmetry depends on the fragmentation function H_1^\perp associated with the Collins effect [1]. The definition of the Collins function is given by [19]

$$\Delta(z, \mathbf{k}_T) = \frac{M}{4P^-} \left\{ D_1(z, \mathbf{k}_T^2) \frac{\not{P}}{M} + H_1^\perp(z, \mathbf{k}_T^2) \frac{\sigma_{\mu\nu} k_T^\mu P^\nu}{M^2} \right\}, \quad (15)$$

where we have only displayed the fragmentation functions that can be present for unpolarized hadron production (D_1 is the ordinary unpolarized fragmentation function). The $\cos(2\phi)$ asymmetry is an azimuthal spin asymmetry in the sense that the asymmetry arises due to the correlation of the transverse spin states of the quark-antiquark pair. On average the quark and antiquark will not be transversely polarized, but for each particular event the transverse spin need not be zero and moreover, the spin states of the quark and antiquark are exactly correlated. Subsequently, the directions of the produced pions are correlated due to the Collins effect and this correlation does not average out after summing over all quark polarization states. The Collins effect correlates the azimuthal angle of the transverse spin of a fragmenting quark with that of the transverse momentum of the produced hadron (both taken around the quark momentum), via a $\sin(\phi)$ distribution of their difference angle ϕ .

One finds for the leading order unpolarized cross section, taking into account both photon and Z -boson contributions [2,3],

$$\begin{aligned} \frac{d\sigma(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2 \mathbf{q}_T} &= \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \sum_{a, \bar{a}} \left\{ K_1^a(y) \mathcal{F}[D_1 \bar{D}_1] \right. \\ &\quad \left. + [K_3^a(y) \cos(2\phi_1) + K_4^a(y) \sin(2\phi_1)] \mathcal{F} \left[\left(2 \hat{\mathbf{h}} \cdot \mathbf{p}_T \hat{\mathbf{h}} \cdot \mathbf{k}_T - \mathbf{p}_T \cdot \mathbf{k}_T \right) \frac{H_1^\perp \bar{H}_1^\perp}{M_1 M_2} \right] \right\}. \end{aligned} \quad (16)$$

The convolutions \mathcal{F} will be discussed below (Eqs. (35) and (37)), but first we will give the definition of the functions $K_i^a(y)$ appearing in this expression and comment on the frame in which this cross section is expressed. As before, a is the flavor index; the functions $K_i^a(y)$ are defined as

$$K_1^a(y) = A(y) [e_a^2 + 2g_V^l e_a g_V^a \chi_1 + c_1^l c_1^a \chi_2] - \frac{C(y)}{2} [2g_A^l e_a g_A^a \chi_1 + c_3^l c_3^a \chi_2], \quad (17)$$

$$K_3^a(y) = B(y) [e_a^2 + 2g_V^l e_a g_V^a \chi_1 + c_1^l c_2^a \chi_2], \quad (18)$$

$$K_4^a(y) = B(y) [2g_V^l e_a g_A^a \chi_3], \quad (19)$$

which contain the combinations of the couplings

$$c_1^j = (g_V^{j2} + g_A^{j2}), \quad (20)$$

$$c_2^j = (g_V^{j2} - g_A^{j2}), \quad (21)$$

$$c_3^j = 2g_V^j g_A^j, \quad (22)$$

where g_V and g_A are the vector and axial-vector couplings to the Z boson. The propagator factors are given by

$$\chi_1 = \frac{1}{\sin^2(2\theta_W)} \frac{Q^2(Q^2 - M_Z^2)}{(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}, \quad (23)$$

$$\chi_2 = \frac{1}{\sin^2(2\theta_W)} \frac{Q^2}{Q^2 - M_Z^2} \chi_1, \quad (24)$$

$$\chi_3 = \frac{-\Gamma_Z M_Z}{Q^2 - M_Z^2} \chi_1. \quad (25)$$

The above cross section is expressed in the following frame. A normalized timelike vector \hat{t} is defined by q (the vector boson momentum) and a normalized spacelike vector \hat{z} is defined by $\tilde{P}^\mu = P^\mu - (P \cdot q/q^2) q^\mu$ for one of the outgoing momenta, for which we choose P_2 (P_i are the hadron momenta),

$$\hat{t}^\mu \equiv \frac{q^\mu}{Q}, \quad (26)$$

$$\hat{z}^\mu \equiv \frac{Q}{P_2 \cdot q} \tilde{P}_2^\mu = 2 \frac{P_2^\mu}{z_2 Q} - \frac{q^\mu}{Q}, \quad (27)$$

The azimuthal angles lie inside the plane orthogonal to \hat{t} and \hat{z} . In particular, ϕ^ℓ gives the orientation of $\hat{l}_\perp^\mu \equiv (g^{\mu\nu} - \hat{t}^\mu \hat{t}^\nu + \hat{z}^\mu \hat{z}^\nu) l_\nu$, the perpendicular part of the lepton momentum l .

In the cross sections we also encounter the following functions of $y = l^-/q^-$, which in the lepton center of mass frame equals $y = (1 + \cos \theta_2)/2$, where θ_2 is the angle of \hat{z} with respect to the momentum of the incoming lepton l :

$$A(y) = \left(\frac{1}{2} - y + y^2 \right) \stackrel{cm}{=} \frac{1}{4} (1 + \cos^2 \theta_2), \quad (28)$$

$$B(y) = y(1 - y) \stackrel{cm}{=} \frac{1}{4} \sin^2 \theta_2, \quad (29)$$

$$C(y) = (1 - 2y) \stackrel{cm}{=} -\cos \theta_2. \quad (30)$$

Since we have chosen P_2 to define the longitudinal direction, the momentum P_1 can be used to express the directions orthogonal to \hat{t} and \hat{z} . One obtains $P_{1\perp}^\mu = g_{\perp}^{\mu\nu} P_{1\nu}$ (see Fig. 1),

$$P_{1\perp}^\mu = -z_1 q_T^\mu = z_1 Q_T \hat{h}^\mu, \quad (31)$$

where we define the normalized vector $\hat{h}^\mu = P_{1\perp}^\mu / |P_{1\perp}|$. The angle ϕ_1 is between \hat{h} and \hat{l}_\perp .

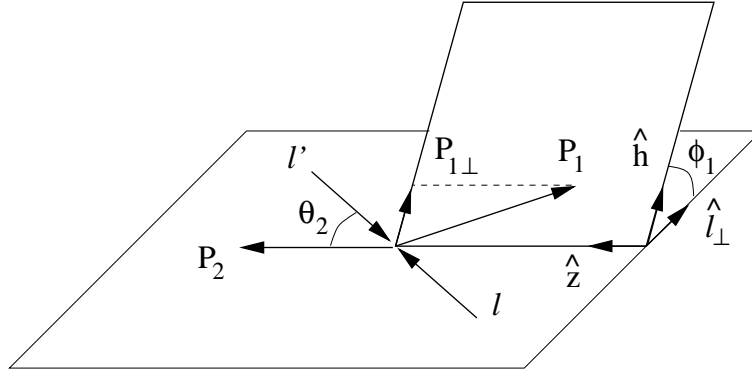


FIG. 1. Kinematics of the annihilation process in the lepton center of mass frame for a back-to-back jet situation. P_2 is the momentum of a hadron in one jet, P_1 is the momentum of a hadron in the opposite jet.

The asymmetry $A(\mathbf{q}_T)$ at the Z mass (ignoring the photon and interference contributions, which can be easily included) is now defined by:

$$\frac{d\sigma(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2 \mathbf{q}_T} \propto \{1 + \cos(2\phi_1) A(\mathbf{q}_T)\}, \quad (32)$$

with

$$A(\mathbf{q}_T) \equiv \frac{\sum_a c_1^e c_2^a B(y) \mathcal{F}[(2 \mathbf{q}_T \cdot \mathbf{p}_T \mathbf{q}_T \cdot \mathbf{k}_T - \mathbf{q}_T^2 \mathbf{p}_T \cdot \mathbf{k}_T) H_1^\perp \overline{H}_1^\perp]}{Q_T^2 M_1 M_2 \sum_b (c_1^e c_1^b A(y) - \frac{1}{2} c_3^e c_3^b C(y)) \mathcal{F}[D_1 \overline{D}_1]}. \quad (33)$$

A confirmation of this asymmetry would confirm the Collins effect without the need of polarization. To avoid repeating irrelevant factors, we will first focus on the numerator and denominator of the term

$$\kappa = \frac{\mathcal{F} \left[(2 \mathbf{q}_T \cdot \mathbf{p}_T \mathbf{q}_T \cdot \mathbf{k}_T - \mathbf{q}_T^2 \mathbf{p}_T \cdot \mathbf{k}_T) H_1^\perp \overline{H}_1^\perp \right]}{Q_T^2 M_1 M_2 \mathcal{F} [D_1 \overline{D}_1]}. \quad (34)$$

In Refs. [2,3] tree level was considered, which means the expressions are valid in the region where the observed transverse momentum is very small compared to the hard scale(s), applicable only in the region of intrinsic transverse momentum. Eq. (34) is a tree level expression if one uses the convolution notation

$$\mathcal{F} [D \overline{D}] \equiv \int d^2 \mathbf{p}_T d^2 \mathbf{k}_T \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T) D^a(z_1, z_1^2 \mathbf{p}_T^2) \overline{D}^a(z_2, z_2^2 \mathbf{k}_T^2), \quad (35)$$

which only involves intrinsic transverse momenta. In order to go beyond this region, we include the Sudakov factor arising from resummed perturbative corrections to the transverse momentum distribution, by considering the approach of Ref. [5,6]. This will extend the range of applicability from the region of intrinsic transverse momentum to the region of moderate \mathbf{q}_T values (still under the restriction that $\mathbf{q}_T^2 \ll Q^2$). This is outlined for the general case in the previous section; here we will apply it to the $\cos(2\phi)$ asymmetry.

Resummation of soft gluons into Sudakov form factors (see e.g. [20]) results in a replacement in Eq. (35) of

$$\delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T) \rightarrow \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{-i\mathbf{b} \cdot (\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T)} e^{-S(b)}, \quad (36)$$

leading to (suppressing the flavor indices)

$$\begin{aligned} \mathcal{F} [D \overline{D}] &\equiv \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} e^{-S(b)} \tilde{D}(z_1, b) \tilde{\overline{D}}(z_2, b) \\ &= \frac{1}{2\pi} \int_0^\infty db b J_0(b Q_T) e^{-S(b)} \tilde{D}(z_1, b) \tilde{\overline{D}}(z_2, b). \end{aligned} \quad (37)$$

The function \tilde{D} denotes the Fourier transform of D . In order to compute the above expression, we will assume a Gaussian transverse momentum dependence for $D_1(z, z^2 \mathbf{k}_T^2)$:

$$D_1(z, z^2 \mathbf{k}_T^2) = D_1(z) R_u^2 \exp(-R_u^2 \mathbf{k}_T^2) / \pi z^2 \equiv D_1(z) \mathcal{G}(|\mathbf{k}_T|; R_u) / z^2. \quad (38)$$

For details see Ref. [19]. Taking the Fourier transform of Eq. (38) yields

$$\tilde{D}_1(z, b^2) = D_1(z) \exp\left(-\frac{b^2}{4R_u^2}\right) / z^2. \quad (39)$$

The numerator in Eq. (34),

$$\begin{aligned} \mathcal{F} [(2 \mathbf{q}_T \cdot \mathbf{p}_T \mathbf{q}_T \cdot \mathbf{k}_T - \mathbf{q}_T^2 \mathbf{p}_T \cdot \mathbf{k}_T) D \overline{D}] &\equiv \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} e^{-S(b)} \\ &\times \int d^2 \mathbf{p}_T d^2 \mathbf{k}_T (2 \mathbf{q}_T \cdot \mathbf{p}_T \mathbf{q}_T \cdot \mathbf{k}_T - \mathbf{q}_T^2 \mathbf{p}_T \cdot \mathbf{k}_T) e^{-i\mathbf{b} \cdot (\mathbf{p}_T + \mathbf{k}_T)} D(z_1, z_1^2 \mathbf{p}_T^2) \overline{D}(z_2, z_2^2 \mathbf{k}_T^2), \end{aligned} \quad (40)$$

cannot be treated exactly like the denominator. A model for the transverse momentum dependence of the function H_1^\perp is needed. In order to get a first estimate, or rather an upper bound for the asymmetry, one might be inclined to assume the maximally allowed function, by saturating the bound satisfied by H_1^\perp [21]

$$|\mathbf{k}_T| |H_1^\perp(z, |\mathbf{k}_T|)| \leq z M_h D_1(z, |\mathbf{k}_T|), \quad (41)$$

producing a $1/|\mathbf{k}_T|$ behavior of $H_1^\perp(z, \mathbf{k}_T)$. However, this is not consistent with the fact that the Collins effect should vanish in the limit $\mathbf{k}_T \rightarrow 0$.

A model by Collins [1] suggests the following transverse momentum dependence. Collins' parameterization for the fragmentation function H_1^\perp is (note that Collins uses the function $\Delta \hat{D}_{H/a} \sim \epsilon_T^{ij} s_{Ti} k_{Tj} H_1^\perp$, where s_T is the transverse spin of the fragmenting quark)

$$\frac{H_1^\perp(z, \mathbf{k}_T^2)}{D_1(z, \mathbf{k}_T^2)} = \frac{2M_C M_h}{\mathbf{k}_T^2 + M_C^2} \text{Im} [A^*(k^2) B(k^2)] \frac{(1-z)}{z}, \quad (42)$$

where M_h is the mass of the produced hadron and M_C is the quark mass that appears in a dressed fermion propagator $i(A(k^2)\not{k} + B(k^2)M_C)/(k^2 - M_C^2)$, the functions A and B are unity at $k^2 = M_C^2$. But for the present purpose, the additional fall-off with $1/\mathbf{k}_T^2$ on top of the Gaussian fall-off is not needed. We will restrict to a Gaussian fall-off (with unknown magnitude) and assume the simple form for $H_1^{\perp a}(z, z^2 \mathbf{k}_T^2) = H_1^{\perp a}(z) \mathcal{G}(|\mathbf{k}_T|; R)/z^2$. Here one should take the radius R to be larger than R_u of the unpolarized function D_1 , such as to satisfy the bound Eq. (41) for all $|\mathbf{k}_T|$. Also, we will assume that the fragmentation functions for both hadrons are Gaussians of equal width, i.e. we take $R_1 = R_2 = R$ and $R_{u1} = R_{u2} = R_u$ and also $M_1 = M_2 = M$.

One finds

$$\int d^2 \mathbf{p}_T d^2 \mathbf{k}_T (2 \mathbf{q}_T \cdot \mathbf{p}_T \mathbf{q}_T \cdot \mathbf{k}_T - \mathbf{q}_T^2 \mathbf{p}_T \cdot \mathbf{k}_T) e^{-i\mathbf{b} \cdot (\mathbf{p}_T + \mathbf{k}_T)} \mathcal{G}(\mathbf{p}_T^2; R) \mathcal{G}(\mathbf{k}_T^2; R) = -\frac{1}{4R^4} [2(\mathbf{q}_T \cdot \mathbf{b})^2 - \mathbf{q}_T^2 \mathbf{b}^2] \exp\left(-\frac{b^2}{2R^2}\right), \quad (43)$$

which after application to Eq. (40) yields

$$\mathcal{F}[(2 \mathbf{q}_T \cdot \mathbf{p}_T \mathbf{q}_T \cdot \mathbf{k}_T - \mathbf{q}_T^2 \mathbf{p}_T \cdot \mathbf{k}_T) D \bar{D}] = \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} \left(-\frac{1}{4R^4}\right) [2(\mathbf{q}_T \cdot \mathbf{b})^2 - \mathbf{q}_T^2 \mathbf{b}^2] e^{-S(b)} \tilde{D}(z_1, b) \tilde{\bar{D}}(z_2, b). \quad (44)$$

It is important to note that the factor $1/R^4$ stems from the intrinsic transverse momentum of the functions $H_1^{\perp}(z, \mathbf{k}_T)$ and therefore, is not dependent on the scale Q (see discussion after Eq. (11)).

In analogy to A_{TT}^V [4], Eq. (34) can be transformed into (keeping in mind that in the end the summation over a, b in numerator and denominator should be performed separately)

$$\kappa = \frac{H_1^{\perp a}(z_1) \bar{H}_1^{\perp a}(z_2)}{4M^4 R^4 D_1^b(z_1) \bar{D}_1^b(z_2)} \mathcal{A}(Q_T) = \frac{H_1^{\perp(1)a}(z_1) \bar{H}_1^{\perp(1)a}(z_2)}{D_1^b(z_1) \bar{D}_1^b(z_2)} \mathcal{A}(Q_T), \quad (45)$$

where for future reference we have also expressed the asymmetry in terms of the function $H_1^{\perp(1)}(z) = z^2 \int d^2 \mathbf{k}_T \mathbf{k}_T^2 / (2M^2) H_1^{\perp}(z, z^2 \mathbf{k}_T^2)$ which occurs frequently in Q_T -weighted cross sections (e.g. [2]); furthermore,

$$\mathcal{A}(Q_T) \equiv M^2 \frac{\int_0^\infty db b^3 J_2(bQ_T) \exp(-S(b_*) - S_{NP}(b))}{\int_0^\infty db b J_0(bQ_T) \exp(-S(b_*) - S_{NP}(b))}. \quad (46)$$

This is similar to the factor $\mathcal{A}(Q_T)$ of Ref. [4], but with the replacement of $J_0(bQ_T) \rightarrow J_2(bQ_T)$ in the numerator. We note that unlike the case of A_{TT}^V investigated in Ref. [4], the present asymmetry does not need to oscillate as a function of Q_T . Rather the vanishing of the numerator after integration over $d^2 \mathbf{q}_T$ is due to the angular integration. Note that κ only depends on Q_T and not on ϕ_1 . Also, we note that this asymmetry has a kinematic zero at $Q_T = 0$, since $\hat{\mathbf{h}}$ cannot be defined in that case (and indeed $J_2(Q_T = 0) = 0$). This is also seen from Ref. [22], where a similar asymmetry factor has been investigated as a function of Q_T .

Here the main focus will be on the factor $\mathcal{A}(Q_T)$, which is a measure for the effect of the Sudakov factors on the asymmetry compared to the tree level result $\exp[-(R^2 - R_u^2)Q_T^2/2] M^2 Q_T^2 R^6/R_u^2$ (cf. Eq. (48)), which is valid only for values of Q_T of the order of the intrinsic transverse momentum.

In the above expression we have introduced the usual cut-off b_{\max} , via $b_* = b/\sqrt{1 + b^2/b_{\max}^2}$ and replaced $\frac{1}{2} b^2/R^2 \rightarrow S_{NP}(b)$, for which we take one of the standard nonperturbative smearing functions, needed to describe the low \mathbf{q}_T region properly. It is important to realize that $S_{NP}(b)$ is introduced only in part to take care of the smearing due to the intrinsic transverse momentum (cf. Eq. (11)), hence one cannot simply equate $\frac{1}{2} b^2/R^2$ with $S_{NP}(b)$. But taking into account the term $\exp(-\frac{1}{2} b^2/R^2)$ in addition to $S_{NP}(b)$ will just produce a change in the coefficient of the b^2 term in $S_{NP}(b)$. To keep the unpolarized cross section unaffected, we will therefore introduce as nonperturbative term $\exp[-S_{NP}(b) + \frac{1}{2} b^2/R^2]$, in order not to count the contribution from intrinsic transverse momentum twice. It is also worth mentioning again that since $R \neq R_u$ (important at tree level), $S_{NP}(b)$ need not be the same in numerator and denominator (much less relevant however, since it affects only g_A, g_B and not g_Q , cf. Eq. (11)) and that in principle, it can depend on z_1 and z_2 , but we will not take into account these refinements.

Here we will take for the nonperturbative Sudakov factor the parameterization of Ladinsky-Yuan (Ref. [23] with $x_1 x_2 = 10^{-2}$),

$$S_{NP}(b) = g_1 b^2 + g_2 b^2 \ln \left(\frac{Q}{2Q_0} \right), \quad (47)$$

with $g_1 = 0.11 \text{ GeV}^2$, $g_2 = 0.58 \text{ GeV}^2$, $Q_0 = 1.6 \text{ GeV}$ and $b_{\text{max}} = 0.5 \text{ GeV}^{-1}$. Part of the results will depend considerably on this choice and this issue will be addressed in detail below. The value $g_1 = 0.11 \text{ GeV}^2$ can be viewed as an intrinsic transverse momentum of $\langle p_T^2 \rangle = 1/R^2 = (220 \text{ MeV})^2$.

The reason we have chosen the parameterization of Ladinsky-Yuan [23], which is fitted to the transverse momentum distribution of W/Z production in pp ($p\bar{p}$) scattering, is that unfortunately there is no nonperturbative Sudakov factor available for $e^+e^- \rightarrow A + B + X$, except for the energy-energy correlation function obtain from low energy data [24]. This is surprising considering the wealth of data from LEP experiments. Moreover, the nonperturbative Sudakov factor in the energy-energy correlation function ($\propto \sum_{A,B} \int dz_1 z_1 dz_2 z_2 Q^2 d\sigma/dQ_T^2$) as fitted in Ref. [24] is not the same as the one in the differential cross section as a function of the lightcone momentum fractions. For related discussions, also relevant for SIDIS, see Refs. [25,26].

Although the nonperturbative Sudakov factors for Drell-Yan and e^+e^- need not be related, the S_{NP} of Ref. [23] can be viewed as a generic one and allows us to study the general features of the Sudakov suppression. We will also investigate the dependence on the nonperturbative Sudakov factor by varying the parameters. At a later stage one can always insert a more appropriate (phenomenologically determined) nonperturbative Sudakov factor into the asymmetry expression Eq. (46). Our results also underline the importance of a good determination of S_{NP} .

In Fig. 2 the asymmetry factor $\mathcal{A}(Q_T)$ is given at the scales $Q = 30 \text{ GeV}$, $Q = 60 \text{ GeV}$ and $Q = M_Z$.

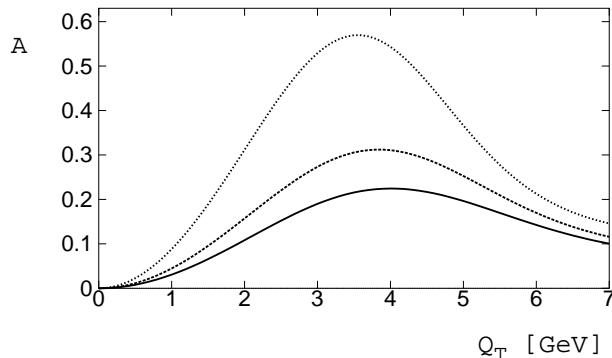


FIG. 2. The asymmetry factor $\mathcal{A}(Q_T)$ (in units of M^2) at $Q = 30 \text{ GeV}$ (upper curve), $Q = 60 \text{ GeV}$ (middle curve) and at $Q = 90 \text{ GeV}$.

The maximum of the asymmetry factor $\mathcal{A}(Q_T)$ at $Q = 30 \text{ GeV}$ ($S_{NP}(b) = 1.41 b^2$), $Q = 60 \text{ GeV}$ ($S_{NP}(b) = 1.81 b^2$) and $Q = M_Z$ ($S_{NP}(b) = 2.05 b^2$) is seen to be 0.57 (at $Q_T \sim 3.6 \text{ GeV}$), 0.31 (at $Q_T \sim 3.8 \text{ GeV}$) and 0.22 (at $Q_T \sim 4 \text{ GeV}$), respectively. One observes that the magnitude of the asymmetry factor goes down with increasing energy and the position of the maximum –and also the average Q_T – shifts to higher values of Q_T .

Now we will discuss the dependence of these results on the choice of b_{max} and S_{NP} . Taking a higher value of b_{max} increases the Gaussian width. The above choice of $b_{\text{max}} = 0.5 \text{ GeV}^{-1}$ can be considered as optimistic ($1/b_{\text{max}}$ is the scale down to which one trusts perturbation theory), enhancing the asymmetry factor somewhat.

The asymmetry factor decreases with increasing Gaussian smearing width in S_{NP} , to which it has a considerable sensitivity. Empirically, we find that if the Gaussian width is reduced by a factor α , then the maximum of the asymmetry increases roughly by α and the corresponding value of Q_T by $\sqrt{\alpha}$. In Fig. 3 this is illustrated for the asymmetry factor $\mathcal{A}(Q_T)$ at $Q = M_Z$. The solid curve is for $S_{NP}(b) = 2.05 b^2$ and the dashed curve for $S_{NP}(b) = 1.37 b^2$, where the latter width is taken from a recent two parameter fit [27].

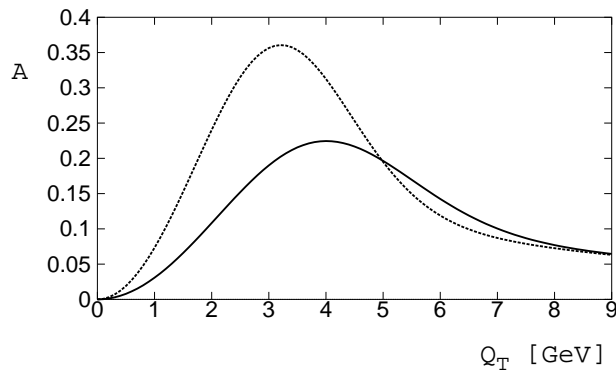


FIG. 3. The asymmetry factor $\mathcal{A}(Q_T)$ (in units of M^2) at $Q = 90$ GeV with $S_{NP}(b) = 2.05 b^2$ (solid curve) and $S_{NP}(b) = 1.37 b^2$ (dashed curve).

The decrease with energy of the maximum of the asymmetry was found not to be very sensitive to changes to the Gaussian width in S_{NP} . We find that the decrease goes as $Q^{-0.9} - Q^{-1.0}$.

So far we have only considered the asymmetry as a function of Q_T , which means the cross section needs to be kept differential in the angle ϕ_1 and the magnitude Q_T of the transverse momentum \mathbf{q}_T . Since the asymmetry does not vanish after integration over Q_T^2 and since the Q_T^2 -integrated cross section has been studied using LEP data [28], we will now consider that case. Note that the integration has to be done in numerator and denominator separately. Since the denominator has no dependence on ϕ_1 (the $\cos(2\phi_1)$ dependence belongs to the numerator) one can in fact integrate over \mathbf{q}_T completely. This corresponds to the inclusive cross section in which soft gluon contributions cancel, hence this integration over the denominator of the asymmetry factor $\mathcal{A}(Q_T)$ yields 1 and has no dependence on Sudakov factors.

The Q_T^2 -integrated result should be compared to tree level, therefore we will first give the tree level expression. The tree level asymmetry is (neglecting the small c_3^a dependent term in the denominator)

$$A^{(0)}(Q_T) = \frac{Q_T^2 R^2 \exp(-R^2 Q_T^2/2)}{4M^2 R_u^2 \exp(-R_u^2 Q_T^2/2)} \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \frac{\sum_a c_2^a H_1^{\perp a}(z_1) \overline{H}_1^{\perp a}(z_2)}{\sum_b c_1^b D_1^b(z_1) \overline{D}_1^b(z_2)}. \quad (48)$$

Note that if one integrates the cross section over Q_T^2 , one should be careful to retain all dependences on Q_T in both numerator and denominator separately. From the above expression we also infer that $\mathcal{A}(Q_T)$ itself should be compared with the tree level quantity $\exp[-(R^2 - R_u^2)Q_T^2/2] M^2 Q_T^2 R^6 / R_u^2$ as mentioned before.

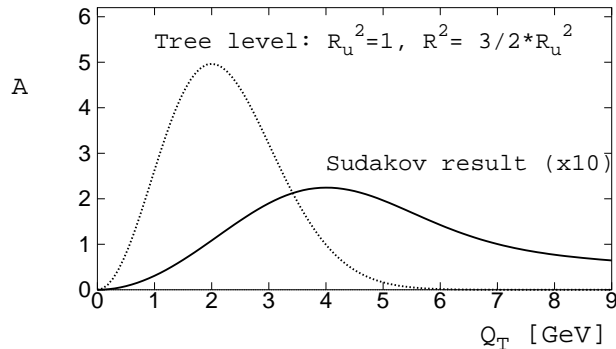


FIG. 4. The asymmetry factor $\mathcal{A}(Q_T)$ (in units of M^2) at $Q = 90$ GeV multiplied by a factor 10 (solid curve) and the tree level quantity (in units of M^2) using $R_u^2 = 1 \text{ GeV}^{-2}$ and $R^2/R_u^2 = 3/2$.

In Fig. 4 we have displayed the comparison of $\mathcal{A}(Q_T)$ at $Q = 90$ GeV and the tree level quantity using the values $R_u^2 = 1 \text{ GeV}^{-2}$ and $R^2/R_u^2 = 3/2$, which were chosen such as to minimize the magnitude. The value $R_u^2 = 1 \text{ GeV}^{-2}$ can be regarded as too small already. We conclude that inclusion of Sudakov factors has the effect of suppressing the tree level result by at least an order of magnitude. This is important to keep in mind when making predictions of transverse momentum dependent azimuthal spin asymmetries based on tree level expressions.

For the particular case of a produced π^+ and π^- , we will make the following simplifying assumptions. We assume $D_1^{u \rightarrow \pi^+}(z) = D_1^{\bar{d} \rightarrow \pi^+}(z)$, $D_1^{d \rightarrow \pi^-}(z) = D_1^{\bar{u} \rightarrow \pi^-}(z)$ and neglect unfavored fragmentation functions like $D^{d \rightarrow \pi^+}(z)$; and similarly for the Collins functions. As a consequence of these assumptions the fragmentation functions can be taken outside the flavor summation and appear as a square (at the average z_1, z_2). After Q_T^2 integration one arrives at a cross section differential in the angle ϕ_1 of \mathbf{q}_T :

$$\frac{d\sigma}{d\Omega d\phi_1} \propto \left(1 + A \left[\frac{H_1^\perp}{D_1} \right]^2 F(y) \cos(2\phi_1) \right), \quad (49)$$

where

$$F(y) = \frac{\sum_{a=u,\bar{d}} c_1^a c_2^a B(y)}{\sum_{b=u,\bar{d}} (c_1^b c_1^b A(y) - \frac{1}{2} c_3^b c_3^b C(y))} \approx \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \frac{\sum_a (g_V^{a^2} - g_A^{a^2})}{\sum_b (g_V^{b^2} + g_A^{b^2})}, \quad (50)$$

which is largest at $\theta_2 = 90^\circ$: $F_{\max} \approx -0.5$ (for a plot of the full factor $F(\theta_2)$ see Ref. [3]).

For the prefactor A one finds at tree level $A^{(0)} = 1/(2M^2 R^2) = \langle \mathbf{p}_T^2 \rangle / (2M^2)$ and one should assume a typical intrinsic transverse momentum squared value, in the range of $\langle \mathbf{p}_T^2 \rangle \approx (200 - 700 \text{ MeV})^2$. For pions this means $A^{(0)} \approx 1 - 12$. This can be compared to $A^{(0)} = 6/\pi$ of Ref. [28], which yields $\langle \mathbf{p}_T^2 \rangle \approx (270 \text{ MeV})^2$ (or $R^2 \approx 13 \text{ GeV}^{-2}$), which seems to be a reasonable value (remember that R^2 must be larger than R_u^2).

After including the Sudakov factors our numerical calculation yields $A = 0.07$ ($S_{NP}(b, Q = M_Z) = 2.05 b^2$), much smaller than the tree level values discussed above. The Gaussian width in S_{NP} is not crucial for this conclusion; if one takes a much smaller width of for instance 1 GeV^2 , then one obtains $A = 0.11$. It has to be emphasized that the width used in $S_{NP}(b, Q)$ should increase with increasing energy.

Our result shows that upon including Sudakov factors one retrieves parton model characteristics (also noted in Ref. [6]), but with transverse momentum spreads that are significantly larger than would be expected from intrinsic transverse momentum (this is supported by the presently available parameterizations of S_{NP} in various processes, see e.g. [25]). This means that a large tree level value for $A^{(0)}$ (e.g. $6/\pi$), would lead to an extracted Collins function that is considerably smaller than if Sudakov factors would have been included. After the nonperturbative Sudakov factor has been obtained from LEP data, one can estimate exactly how much. For our choice of $S_{NP}(b, Q = M_Z) = 2.05 b^2$, the effect on the magnitude of the Collins function is an additional factor $\sqrt{6/(0.07\pi)} \approx 5$ compared to the extraction using the tree level expression.

We conclude that the Sudakov factors produce a strong suppression compared to the tree level result. A tree level analysis applied at $Q = M_Z$ is expected to overestimate the asymmetry at least by an order of magnitude and therefore, it will underestimate the Collins function significantly. Hence, Sudakov factors should be taken into account when extracting the Collins function from LEP data or in general, from e^+e^- data obtained at high values of \sqrt{s} .

IV. COLLINS ASYMMETRY IN SIDIS

The Collins function H_1^\perp originally was shown to lead to a single spin azimuthal asymmetry in semi-inclusive DIS [1]. This asymmetry has received much attention, since it would provide an additional way of accessing the transversity function h_1 . A preliminary determination of the Collins asymmetry from SMC data has been performed [29], yielding an asymmetry for π^+ production of $11\% \pm 6\%$. Also, the $\sin \phi$ azimuthal asymmetry as recently measured by the HERMES Collaboration [30] might be related to the Collins asymmetry, providing further indication that the Collins function is nonzero. Measurements by HERMES and COMPASS are expected to provide more conclusive information on the Collins effect and its magnitude. Since these experiments are performed at different energies, it is important to know the Q^2 dependence of the asymmetry. Here we will investigate the Q^2 dependence of the transverse momentum (Q_T) distribution of the asymmetry.

From Ref. [31] we extract the expression for this single spin asymmetry (extended to include contributions from Z -boson exchange; the expressions for W -boson exchange can be found in Ref. [31]):

$$\frac{d\sigma(\ell H \rightarrow \ell' h X)}{dx dz dy d\phi_\ell d^2 \mathbf{q}_T} = \frac{2\alpha^2 x z^2 s}{Q^4} \sum_{a,\bar{a}} \left\{ K_1^a(y) \mathcal{F}[f_1 D_1] - |\mathbf{S}_T| K_3^a(y) \sin(\phi_h^\ell + \phi_S^\ell) \mathcal{F}\left[\hat{\mathbf{h}} \cdot \mathbf{k}_T \frac{h_1 H_1^\perp}{M_h}\right] + \dots \right\}, \quad (51)$$

where \mathbf{S}_T is the transverse spin of the incoming hadron H . The couplings $K_1^a(y)$ and $K_3^a(y)$ are of the same form as before, except that now

$$A(y) = \left(1 - y + \frac{1}{2}y^2\right), \quad (52)$$

$$B(y) = -(1 - y), \quad (53)$$

$$C(y) = -y(2 - y), \quad (54)$$

and $y = (P \cdot q)/(P \cdot l) \approx q^-/l^-$ (l is the momentum of the beam lepton). Also, since Q^2 is now space-like, the width Γ_Z can be ignored:

$$\chi_1 = \frac{1}{\sin^2(2\theta_W)} \frac{Q^2}{Q^2 + M_Z^2}, \quad (55)$$

$$\chi_2 = (\chi_1)^2. \quad (56)$$

The azimuthal angle ϕ_h^ℓ (ϕ_S^ℓ) around the three-momentum of the virtual boson is between the lepton's three-momentum and the outgoing hadron's three-momentum (transverse spin), cf. Ref. [31].

If we write the cross section as

$$\frac{d\sigma(\ell H \rightarrow \ell' h X)}{dx dz dy d\phi_\ell d^2\mathbf{q}_T} \propto \{1 - |\mathbf{S}_T| \sin(\phi_h^\ell + \phi_S^\ell) A(\mathbf{q}_T)\}, \quad (57)$$

the asymmetry analyzing power is given by

$$A(\mathbf{q}_T) \equiv \frac{\sum_a K_3^a(y) \mathcal{F}[\mathbf{q}_T \cdot \mathbf{k}_T h_1 H_1^\perp]}{Q_T M_h \sum_a K_1^a(y) \mathcal{F}[f_1 D_1]}. \quad (58)$$

Assuming again Gaussian transverse momentum dependence and including Sudakov factors one arrives at

$$A(\mathbf{q}_T) = \frac{\sum_a K_3^a(y) h_1^a(x) H_1^{\perp a}(z)}{2M_h^2 R^2 \sum_b K_1^b(y) f_1^b(x) D_1^b(z)} \mathcal{A}(Q_T), \quad (59)$$

where R^2 is the Gaussian width of H_1^\perp and the asymmetry factor $\mathcal{A}(Q_T)$ is defined as

$$\mathcal{A}(Q_T) \equiv M_h \frac{\int_0^\infty db b^2 J_1(bQ_T) \exp(-S(b_*) - S_{NP}(b))}{\int_0^\infty db b J_0(bQ_T) \exp(-S(b_*) - S_{NP}(b))}. \quad (60)$$

This should be compared with Eq. (46) of the previous section. In Fig. 5 the asymmetry factor $\mathcal{A}(Q_T)$ is given at the scales $Q = 30$ GeV, $Q = 60$ GeV and $Q = M_Z$.

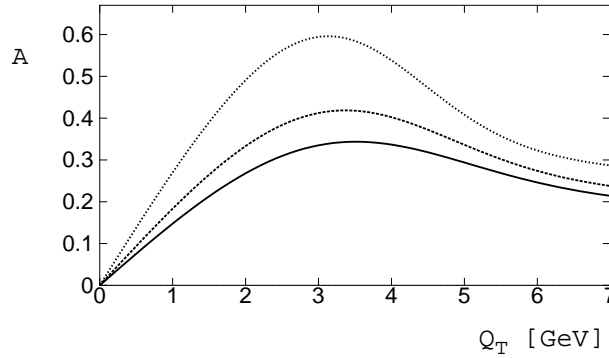


FIG. 5. The asymmetry factor $\mathcal{A}(Q_T)$ (in units of M_h) at $Q = 30$ GeV (upper curve), $Q = 60$ GeV (middle curve) and at $Q = 90$ GeV.

The maximum of the asymmetry factor $\mathcal{A}(Q_T)$ at $Q = 30$ GeV, $Q = 60$ GeV and $Q = M_Z$ is seen to be 0.60 (at $Q_T \sim 3.1$ GeV), 0.42 (at $Q_T \sim 3.4$ GeV) and 0.34 (at $Q_T \sim 3.5$ GeV), respectively. Again we note that the magnitude of the asymmetry factor goes down with increasing energy and the position of the maximum shifts to higher values of Q_T .

As before we have used the nonperturbative Sudakov factor by Ladinsky and Yuan, Eq. (47), in the absence of a well-established S_{NP} for SIDIS (cf. Ref. [25,26]). But the results obtained here can nevertheless be viewed as generic, only changes in the specific numbers are expected. Like for the previous $\cos(2\phi)$ asymmetry, the asymmetry factor decreases with increasing Gaussian smearing width. Empirically we find that if the Gaussian width in S_{NP} is reduced by a factor α , then both the maximum of the asymmetry factor and the corresponding value of Q_T increase roughly by $\sqrt{\alpha}$. However, the decrease with energy of the maximum of the asymmetry is not very sensitive to changes in this Gaussian width. We find that the decrease goes as $Q^{-0.5} - Q^{-0.6}$, which is considerably slower than for the $\cos(2\phi)$ asymmetry. In the next section we will discuss the comparison between different asymmetries in more detail.

We will now compare these results to the tree level expression for this Collins asymmetry. At tree level the convolution \mathcal{F} (where w denotes a weight function) is given by

$$\mathcal{F}[w(\mathbf{p}_T, \mathbf{k}_T) f D] \equiv \int d^2\mathbf{p}_T d^2\mathbf{k}_T \delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T) w(\mathbf{p}_T, \mathbf{k}_T) f^a(x, \mathbf{p}_T^2) D^a(z, z^2 \mathbf{k}_T^2). \quad (61)$$

If Gaussian transverse momentum distribution and fragmentation functions are assumed, one obtains at tree level

$$\mathcal{A}^{(0)}(Q_T) = \frac{Q_T R^2 \exp(-R^2 Q_T^2/2)}{2M_h R_u^2 \exp(-R_u^2 Q_T^2/2)} \frac{\sum_a K_3^a(y) h_1^a(x) H_1^{\perp a}(z)}{\sum_b K_1^b(y) f_1^b(x) D_1^b(z)}. \quad (62)$$

Therefore, one should compare $\mathcal{A}(Q_T)$ with the tree level quantity $\exp[-(R^2 - R_u^2)Q_T^2/2] M_h Q_T R^4/R_u^2$.

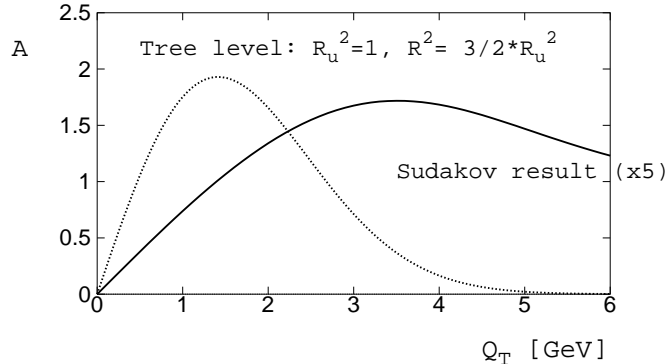


FIG. 6. The asymmetry factor $\mathcal{A}(Q_T)$ (in units of M_h) at $Q = 90$ GeV multiplied by a factor 5 (solid curve) and the tree level quantity (in units of M_h) using $R_u^2 = 1 \text{ GeV}^{-2}$ and $R^2/R_u^2 = 3/2$.

In Fig. 6 we have displayed the comparison of $\mathcal{A}(Q_T)$ at $Q = 90$ GeV and the tree level quantity using again the values $R_u^2 = 1 \text{ GeV}^{-2}$ and $R^2/R_u^2 = 3/2$, which minimize the maximum asymmetry value. As before we conclude that inclusion of Sudakov factors has the effect of suppressing the tree level result. It is also clear that for the Collins asymmetry (which arises from a single Collins effect) the Sudakov suppression is less severe than for the case of the $\cos(2\phi)$ asymmetry (which depends on the Collins effect squared). One also observes a pronounced increase of the average Q_T upon inclusion of Sudakov factors. The difference to tree level becomes even more pronounced as the choice of R_u^2 is increased to larger values more appropriate for a tree level analysis. Hence, also for transverse momentum dependent azimuthal spin asymmetries in SIDIS one needs to include Sudakov factors in order to extract distribution and fragmentation functions reliably.

We now turn to a more general discussion of the different types of transverse momentum dependent azimuthal spin asymmetries, categorized by different transverse momentum weights and angular dependences. The comparisons are most cleanly done for the asymptotic Q^2 case.

In the previous sections we have studied asymmetries with different types of weights, namely $2\hat{\mathbf{h}}\cdot\mathbf{p}_T\hat{\mathbf{h}}\cdot\mathbf{k}_T - \mathbf{p}_T\cdot\mathbf{k}_T$ and $\hat{\mathbf{h}}\cdot\mathbf{k}_T$. The former (and also $\mathbf{p}_T\cdot\mathbf{k}_T$ in the asymmetry A_{TT}^V of Ref. [4]) is typical of a double transverse spin asymmetry. It has two powers of transverse momentum and is therefore expected to decrease faster than the (single spin) asymmetries which have weights with a single power. The reason for this difference in decrease is that the asymmetries, which are convolutions of transverse momentum distributions, are largest if these distributions are large and have a large overlap. The distributions are largest at small \mathbf{p}_T and \mathbf{k}_T and the overlap is largest at small transverse momentum \mathbf{q}_T . However, the explicit powers of transverse momentum in the weights tend to suppress the region of small transverse momentum. More powers of transverse momentum in the weight thus implies more suppression. The power n of b^n in the numerator compared to the denominator is a measure of this effect. A weight proportional to $\mathbf{p}_T^i\mathbf{k}_T^j$ will lead to $n = 2$ (cf. Eq. (46)), whereas \mathbf{p}_T^i or \mathbf{k}_T^j will lead to $n = 1$ (cf. Eq. (60)). Larger n means, larger suppression and since the transverse momentum distributions broaden and decrease in magnitude with increasing energy, the convolution also decreases. Asymmetries with $n = 1$ will fall off slower with energy and the Sudakov suppression is less severe. This natural expectation is clearly observed in the numerical analysis of the examples investigated here and in Ref. [4].

It is interesting to see what is the asymptotic behavior of the Sudakov suppression. As is usually done, we will assume that at high values of Q the nonperturbative factor is irrelevant (true only asymptotically) and consider asymmetry factors of the form

$$\mathcal{A}_{n,m}(Q_T) \equiv M^n \frac{\int_0^\infty db^2 b^n J_m(bQ_T) \exp(-S(b))}{\int_0^\infty db^2 J_0(bQ_T) \exp(-S(b))}. \quad (63)$$

The asymptotic behavior of the denominator at $Q_T = 0$ (relevant for the unpolarized differential cross section) has been studied before [32,7] by means of a saddle point approximation. Unfortunately, such a saddle point approximation cannot be applied to the ratios $\mathcal{A}_{n,m}(Q_T)$ for $m \neq 0$ and $Q_T \neq 0$, since one cannot take $J_m(bQ_T) \approx J_m(b_{SP}Q_T)$ to deduce the Q^2 dependence. For instance, since $b_{SP}Q_T$ is in general not small, i.e. $\ll m$ (even under the assumption $Q_T^2 \ll Q^2$), no approximation to the Bessel functions can be used in the region of interest (i.e., around the value of Q_T where the asymmetry is maximal).

In Ref. [4] the asymmetry factor $\mathcal{A}_{2,0}(Q_T)$ arises, which does allow for a straightforward saddle point approximation at $Q_T = 0$, where the asymmetry is maximal. Therefore, we simply assume that the weight $2\hat{\mathbf{h}}\cdot\mathbf{p}_T\hat{\mathbf{h}}\cdot\mathbf{k}_T - \mathbf{p}_T\cdot\mathbf{k}_T$ has similar behavior as $\mathbf{p}_T\cdot\mathbf{k}_T$. From our numerical studies we conclude that the maximum of $\mathcal{A}_{2,2}(Q_T)$ actually decreases faster with energy than the maximum of $\mathcal{A}_{2,0}(Q_T)$ (at $Q_T = 0$). Moreover, for the asymptotic behavior of $\mathcal{A}_{n,0}(Q_T = 0)$ one can obtain an analytic expression. First we would like to comment on the saddle point approximation of expressions of the form

$$\int_0^\infty db^2 b^n \exp(-S(b)) = \int \frac{d\ln(b^2\Lambda^2)}{(\Lambda^2)^{1+\frac{n}{2}}} \exp\left\{\left(1+\frac{n}{2}\right)\ln(b^2\Lambda^2) - S(b)\right\}. \quad (64)$$

As before we will keep only the leading term in the expansion of $A(\alpha_s(\mu))$ and $\alpha_s(\mu)/\pi = 1/(\beta_1 \ln(\mu^2/\Lambda^2))$:

$$S(b, Q) = \int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \frac{C_F}{\beta_1 \ln(\mu^2/\Lambda^2)} \ln \frac{Q^2}{\mu^2}. \quad (65)$$

The integral Eq. (64) has a saddle point at

$$b_{SP} = \frac{b_0}{\Lambda} \left(\frac{Q}{\Lambda}\right)^{-C_F/[C_F+(1+\frac{n}{2})\beta_1]}. \quad (66)$$

For $n = 0$ we retrieve the power -0.41 of Refs. [32,7]. For general n one finds in the saddle point approximation (defining $\gamma_n = -C_F/[C_F+(1+\frac{n}{2})\beta_1]$)

$$\int_0^\infty db^2 b^n \exp(-S(b)) \stackrel{SP}{\propto} \left(\frac{b_0^2}{\Lambda^2}\right)^{(1+\frac{n}{2})} \left(\frac{Q^2}{\Lambda^2}\right)^{(1+\frac{n}{2})\gamma_n + \frac{C_F}{\beta_1}(1+\gamma_n+\ln(-\gamma_n))}. \quad (67)$$

In this way one finds for the denominator of $\mathcal{A}_{2,0}(Q_T = 0)$ an approximate power behavior of $(Q^2)^{-0.94}$ and for the numerator $(Q^2)^{-0.62}$, when one takes $C_F = 4/3$ and $\beta_1 = 23/12$ (corresponding to 5 flavors, but 6 flavors

only makes a few percent difference). The approximated asymmetry factor $\mathcal{A}_{2,0}(Q_T = 0)$ then has an asymptotic power behavior of $(Q^2)^{-0.32}$, which is a similar decrease as found numerically in Ref. [4] and apparently holds also for very large values of Q . At lower energies, the saddle point is not very pronounced and the introduction of the Gaussian smearing and b_* will have an effect on the approximation, hence we view this agreement as a coincidence. Of course, the saddle point approximation should become better with increasing Q^2 .

For the ratios $\mathcal{A}_{n,0}(Q_T = 0)$ one can also derive an analytic expression for the asymptotic Q^2 dependence:

$$\mathcal{A}_{n,0}(Q_T = 0) = M^n \frac{\int_0^\infty db^2 b^n \exp(-S(b))}{\int_0^\infty db^2 \exp(-S(b))} = c_n \left(\frac{M^2 b_0^2}{\Lambda^2} \right)^{\frac{n}{2}} \left(\frac{Q^2}{\Lambda^2} \right)^{\frac{C_F}{\beta_1} \ln c_n}, \quad (68)$$

where $c_n = (1 + C_F/\beta_1)/(1 + n/2 + C_F/\beta_1)$. For $n = 2$ this yields $(Q^2)^{-0.32}$ for 5 flavors, which shows that the saddle point approximation is an extremely good approximation (the power of Q^2 differs by $\mathcal{O}(10^{-9})$). One now expects that the maximum value of $\mathcal{A}_{1,1}(Q_T)$ (appearing in the Collins asymmetry) to fall off at least as fast as $\mathcal{A}_{1,0}(Q_T = 0)$ (for all Q), which asymptotically goes as $(Q^2)^{-0.18}$. We note that the introduction of the Gaussian smearing and b_* have a considerable effect on the approximation at lower energies as our numerical studies demonstrate (in the studied range of 10 – 100 GeV we found approximately $1/Q$ for $\max(\mathcal{A}_{2,2}(Q_T))$ and $1/\sqrt{Q}$ for $\max(\mathcal{A}_{1,1}(Q_T))$). Our main conclusion is that for very high Q transverse momentum dependent azimuthal spin asymmetries will fall off as a fractional power of $1/Q$ and that the behavior (or an upper bound to it) can approximately be found by looking at the power of b in the integrand.

Finally, we want to mention that the increase of the Sudakov suppression with energy provides a solution to the following problem. In general, azimuthal spin asymmetries in collinear configurations, where partons are collinear to the parent hadrons, are suppressed by explicit powers of the hard energy scales. The transverse momentum factors in the weights have to be generated in the hard scattering part and for this one pays a price in terms of inverse powers of the hard scale. The collinear case can be viewed as averaged over the transverse momenta, relevant for \mathbf{q}_T -integrated cross sections, which receives its main contribution from the region of small transverse momentum. On the other hand, in the case of \mathbf{q}_T -dependent differential cross sections at small \mathbf{q}_T (compared to Q), small parton transverse momentum needs to be included (one deals with nearly collinear partons therefore). In this case there need not be an explicit power suppression to generate similar azimuthal spin asymmetries (the average transverse momentum is now a scale in the problem, so dimensionless ratios using Q_T rather than Q can be formed). This seems counterintuitive, since by considering a differential cross section for a nearly collinear configuration, one can obtain information without power suppression, that is power suppressed in the \mathbf{q}_T -integrated cross section, which receives its main contribution from the region of small transverse momentum also (for an example, see Ref. [33]). But we find that instead of explicit power suppression the presence of Sudakov factors gives rise to partial power suppression. In this sense the power suppression is effectively replaced by a Sudakov suppression. This means that the \mathbf{q}_T -dependent azimuthal spin asymmetries vanish with increasing energy, as do their \mathbf{q}_T -integrated counterparts. Hence, departing from the collinear configuration (that is, by including small transverse momenta) does not allow one to measure azimuthal spin asymmetries in (differential) cross sections at arbitrary high energies.

VI. CONCLUSIONS

In this article we have shown by quantitative examples how transverse momentum dependent azimuthal spin asymmetries are suppressed by Sudakov factors, in the region where the transverse momentum is much smaller than the large energy scale Q^2 . Physically, the Sudakov suppression stems from broadening of the transverse momentum distribution due to recoil from soft gluon radiation and the suppression increases with energy. This implies that tree level estimates of transverse momentum dependent azimuthal spin asymmetries tend to overestimate the magnitudes and increasingly so with rising energy.

The size and Q^2 dependence of the Sudakov suppression have been studied numerically for two such asymmetries, both arising due to the Collins effect. The size of the suppression (compared to tree level) depends considerably on the nonperturbative Sudakov factor that must be determined from experiment; however, the Q^2 dependence of the suppression turned out to be much less sensitive to the nonperturbative input.

We observe that in general the larger the power n of the transverse momentum in the weight of an asymmetry, the larger the suppression. For the Collins effect driven $\cos(2\phi)$ asymmetry in electron-positron annihilation into two almost back-to-back pions ($n = 2$), the Sudakov suppression was numerically found to be approximately $1/Q$ for the maximum of the asymmetry and an upper bound for the asymptotic behavior was found to be $1/Q^{0.6}$. For the Collins effect single spin asymmetry in semi-inclusive deep inelastic scattering ($n = 1$), the

Sudakov suppression was numerically found to be approximately $1/\sqrt{Q}$ for the maximum of the asymmetry and an upper bound for the asymptotic behavior was found to be $1/Q^{0.4}$. For the maximum of the asymmetries of the type $\mathcal{A}_{n,0}(Q_T)$, the asymptotic Q^2 dependence could be calculated analytically. This provides upper bounds on the fall-off of $\mathcal{A}_{n,m}(Q_T)$, with $m \neq 0$.

Since our results depend on the input for the nonperturbative Sudakov factors S_{NP} , which is not (well) determined for the above processes, the numerical conclusions about the size and Q^2 dependence of the suppression should be viewed as generic, not as precise predictions. Therefore, we would like to stress the need for an extraction of the nonperturbative Sudakov factor from the process $e^+e^- \rightarrow A + B + X$ (for any two, almost back-to-back hadrons A and B) and from SIDIS. Considering the wealth of data from the LEP and HERA experiments this should pose no problem.

The Sudakov suppression of the transverse momentum distribution of azimuthal spin asymmetries is significant already for Q values in the range of $10 - 100$ GeV as can be concluded from comparison to tree level. In single spin asymmetries the magnitude of the suppression is less severe than in double spin asymmetries and in both cases a pronounced shift of the average Q_T to higher values is observed. We conclude that it is essential to take into account Sudakov factors in transverse momentum dependent azimuthal spin asymmetries.

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